GENERALIZATION OF THE CHARACTERISTICS OF ELECTRIC ARCS (SURVEY)

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I. SELECTION OF CRITERIA

Electric arcs are now used in various kinds of gas heaters and plasma generators employed both in scientific research and in industry. Recent progress in mastering the region of high temperatures, extending to thousands and even tens of thousands of degrees, owes much to the plasma generator.

Hypersonic flight in planetary atmospheres can be simulated by means of plasma generators, which are also used to investigate the physical properties of gases and heat and mass transfer processes at high temperatures and flow velocities.

However, the industrial applications of electric-arc heating are no less important. Refractory processing, the deposition of heat-resistant and anticorrosion coatings, the creation and spheroidization of ultradispersed powders, the thermal reduction of various metals from ores, plasma refining to eliminate undesirable impurities, nonoxidative heating, various chemical syntheses (to obtain acetylene, cyanogen, nitric oxide, etc.)—this is only a very incomplete list of the important processes now in use or shortly to be introduced. All this means that electric-arc heating already occupies an important place in science and industry, while its role is continuously increasing with the development of high-temperature technology. Accordingly, the methods of analyzing and designing electric arc heaters are of considerable interest.

The processes that take place in an arc heater are complex and varied. The very intense conversion of electrical power into thermal energy is achieved in small volumes and is accompanied by the heating of the gas to high temperatures calculated in thousands and tens of thousands of degrees, with all the consequences that this implies: thermal dissociation and ionization of the particles, interaction of the charges with the electromagnetic field, high temperature and concentration gradients associated with mass and energy diffusion fluxes, intensification of the rates of chemical reactions, etc.

In general, this complex of interrelated processes can be described by a system of equations that includes the laws of conservation of mass, momentum, charge and energy, the laws of the electromagnetic field and transport laws, and the relations between the thermodynamic and kinetic properties and the parameters of state of the system. Even if this system is simplified as far as possible by discarding the less important processes, there still remains a quite complicated set of equations, whose solution involves considerable mathematical difficulties. For example, if we neglect friction, mass diffusion fluxes, volume radiation, and chemical reactions and take the diffusional energy transport into account in the general coefficient of thermal conductivity, for the steady-state regime we obtain the following system of equations:

$$\rho\,(\mathbf{w}\,\mathrm{grad})\,\mathbf{w} = -\,\mathrm{grad}\,P + [\mathbf{jB}], \tag{1}$$

$$\mathrm{div}\,\rho\,\mathbf{w} = 0, \tag{2}$$

$$\mathrm{div}\,\mathbf{j} = 0, \tag{3}$$

$$\rho\,\mathbf{w}\,\mathrm{grad}\left(h + \frac{\mathbf{w}^2}{2}\right) = \mathrm{div}\,(\lambda\,\mathrm{grad}\,T) + \mathbf{jE}, \tag{4}$$

$$\mathrm{rot}\,\mathbf{B} = \mu_0\,\mathbf{j}, \tag{5}$$

$$\mathbf{j} = \sigma\,\mathbf{E}. \tag{6}$$

To close the system, it is necessary to introduce expressions relating the density, enthalpy, and the electrical and thermal conductivities to the temperature, pressure, and the nature of the gas.

Even for laminar flows the solution of this simplified system is a very complicated process, since apart from the mathematical difficulties there is also the problem of determining the properties of the plasma, which for many gas mixtures of practical importance are not known. However, laminar motion in electric arc heaters is realized only in certain cases. In most such equipment the flows are turbulent and an analytic approach to the solution of the problem is not sufficient. In this case, we do not have a closed system of equations which would make it possible to reduce the solution to a purely mathematical problem.

At the same time, the experimental investigation of electric arc equipment is also rather complicated and expensive, especially at high powers. Clumsy electrical, gas and water supply systems, complicated experimental discharge chambers, which, moreover, often break down in the first unsuccessful experiment, the large number of independent variables that must be varied during the experiment, all these make it difficult to accumulate the information needed for an accurate analysis.

Thus, the problem arises of generalizing the experimental data in order as far as possible to save time and money that would otherwise be spent on research.

The most effective means of generalizing the experimental data is to employ the theory of similarity. Generalizations of this kind applicable to electric arcs began to be developed in the Soviet Union in 1962 and a series of initial reports were published in the period 1964–1965 [1–3, 6, 8].

The similarity criteria can be obtained by dimensional analysis or by reducing the equations to dimensionless form. In particular, from the system of equations presented above it is possible to obtain a series of generalized variables.

From the equation of motion the following criteria are obtained:

$$\Pi_1 = \frac{P_0}{\rho_0 w_0^2}; \quad \Pi_2 = \frac{j_0 BL}{\rho_0 w_0^2}.$$

The first criterion is analogous to the Euler number, but as distinct from the latter not the pressure drop but some absolute value P_0 is taken as the scale of reference, since in this case the physical properties of the gas depend to a considerable degree on the pressure. It characterizes the relationship between pressure and inertia forces and appears when these systems are commensurable.

The second criterion characterizes the relationship between the electromagnetic and inertia forces. It plays a certain role in the presence of external magnetic fields with a known induction B. At considerable currents, the intrinsic magnetic field (self-field) begins to exert an effect. Then, using Eq. (5), we can reduce the criterion Π_2 to the form

$$\Pi_3 = \frac{j_0^2 L^2 \mu_0}{\rho_0 w_0^2}.$$

However, if both the external and the self-fields are important, then both Π_2 and Π_3 must be taken simultaneously into account. Instead of Π_3 it is also possible to use the dimensionless complex, obtained from equation (5):

$$\Pi_4 = \frac{\mu_0 j_0 L}{B},$$

characterizing the relationship between the induction of the self-field and the external field.

Equations (2) and (3), having only one term each and reflecting the continuity of the mass fluxes and the currents in the steady state, do not give criteria.

From the energy equation it is possible to obtain three criteria:

$$\Pi_5 = \frac{\rho_0 w_0 h_0}{L j_0 E_0}; \quad \Pi_6 = \frac{\rho_0 w_0^3}{L j_0 E_0}; \quad \Pi_7 = \frac{\lambda_0 T_0}{L^2 j_0 E_0}$$

The first of these three dimensionless complexes reflects the process of conversion of electricity into thermal energy of the gas flowing through the arc. Since plasma generators are designed specifically to heat gas, the criterion Π_5 must be one of the principal dimensionless arguments.

A sharp increase in the temperature of the heated gas and a corresponding fall in density at a given flow rate and channel cross section are associated with a considerable increase in gas velocity. The process of conversion of electrical power into energy of directional motion is characterized by the criterion Π_6. This dimensionless argument may be especially important for the heaters used in hypersonic wind tunnels, especially if the electric arc burns in an accelerating nozzle.

The following complex reflects the process of removal of Joule heat by conduction. Its influence may be especially important in equipment with low efficiency and low plasma velocities (for example, in long water-stabilized arcs, in arcs stabilized by segmented diaphragms at low gas flow rates, etc.).

Expression (6) determines the relation between the scale values σ_0, E_0, and j_0. In practice, it is necessary to deal not with the current density, but with the current. They are related through the characteristic dimension $I \sim j_m L^2$. In this case it is usual for one quantity—either the current or the voltage—to be specified with the other left to be determined. By assigning the value of the current, it is possible to set $j_m = j_0$. Then, since E/E_0 depends on other criteria, from (6) we obtain a dimensionless number that can be used as a function of the generalized current-voltage characteristic:

$$\Pi_8 = \frac{\sigma_0 E L^2}{I}.$$

The dimensionless arguments Π_1–Π_7, respectively, become

$$\Pi_1' = \Pi_1 = \frac{P_0}{\rho_0 w_0^2}; \quad \Pi_2' = \frac{IB}{\rho_0 w_0^2 L}; \quad \Pi_3' = \frac{\mu_0 I^2}{\rho_0 w_0^2 L^2};$$
$$\Pi_4' = \frac{\mu_0 I}{BL}; \quad \Pi_5' = \frac{\rho_0 w_0 h_0 \sigma_0 L^3}{I^2}; \quad \Pi_6' = \frac{\rho_0 \sigma_0 w_0^3 L^3}{I^2};$$
$$\Pi_7' = \frac{\lambda_0 \sigma_0 T_0 L^2}{I^2}.$$

These complexes are convenient to use when the current and the gas flow velocity are given. In many cases, however, we are given not the flow velocity but the flow rate of the gas. Setting $G \sim L^2 \rho_0 w_0$, we obtain the new system of dimensionless quantities:

$$\Pi_1'' = \frac{\rho_0 P_0 L^4}{G^2}; \quad \Pi_2'' = \frac{\rho_0 IBL^3}{G^2}; \quad \Pi_3'' = \frac{\mu_0 \rho_0 I^2 L^2}{G^2};$$
$$\Pi_4' = \frac{\mu_0 I}{BL}; \quad \Pi_5'' = \frac{\sigma_0 h_0 G L}{I^2}; \quad \Pi_6'' = \frac{G^3 \sigma_0}{\rho_0^2 I^2 L^3};$$
$$\Pi_7'' = \Pi_7' = \frac{\lambda_0 \sigma_0 T_0 L^2}{I^2}.$$

Instead of the criteria obtained it is sometimes convenient, for reducing the experimental data, to employ their individual combinations, which are also criteria. These include:

$$\Pi_2''/\Pi_1'' = \Pi_9'' = \frac{IB}{P_0 L}; \quad \Pi_3''/\Pi_1'' = \Pi_{10}'' = \frac{\mu_0 I^2}{P_0 L^2};$$

$$\Pi_5''/\Pi_7'' = \mathrm{Pe} = \mathrm{Re}\,\mathrm{Pr} = \frac{Gc_{p_0}}{\lambda_0 L};$$

$$\Pi_7''/\Pi_6'' = \Pi_{11}'' = \frac{\lambda_0 T_0 \rho_0^2 L^5}{G^3};$$

$$\Pi_5''/\Pi_6'' = \Pi_{12}'' = \frac{h_0 \rho_0^2 L^4}{G^2}.$$

The criterion Π_9'' expresses the relationship between the electromagnetic and pressure forces. It is convenient when the forces of interaction between the external magnetic field and the current cause a significant pressure drop. The complex Π_{10}'' reflects the "pinch effect"—the increase in pressure due to compression by the magnetic field of the current. The Pe number is convenient to use if the gas is heated at the periphery of the arc column as a result of removal of energy from the arc by heat conduction. If under such conditions the gas is accelerated, the criterion Π_{11}'' is used.

The criterion Π_{12}'' characterizes the conversion of thermal energy into energy of translational motion associated with the acceleration of the gas. Combinations of dimensionless complexes are sometimes convenient for reducing the experimental data. For example, if the criteria Π_5 and Π_7 are both important, the analysis can be accelerated by using one of these complexes and the Pe number.

The criteria contain the scale values of the physical properties P_0, T_0, ρ_0, h_0, σ_0, λ_0. However, not all of them can be selected arbitrarily. The relationship between them is determined by the dependence of the plasma properties on P and T. Therefore, if we specify the values of P_0 and T_0, we can also find the scales of reference ρ_0, h_0, σ_0, λ_0.

The system of dimensionless quantities Π_1–Π_7 is, however, incomplete, since only the basic laws are described by system (1)–(6). To specify the problem, it is necessary to define the boundary conditions, from which it is possible to obtain a further series of parametric criteria. In formulating the boundary conditions we encounter considerable difficulties. Of course, if the temperature, pressure and velocity at the boundaries are characterized only by some individual value and this given quantity can be combined with a scale of reference, then no additional parametric criteria, apart from the geometric relations characterizing the multidimensionality of space L_i/L, appear. But if a series of characteristic values are given (for example, the temperatures at the boundaries of the system T_i, the pressures and velocities in various sections P_i, w_i), then among the dimensionless arguments it is necessary to include the parametric criteria

$$\frac{T_i}{T_0};\quad \frac{P_i}{P_0};\quad \frac{w_i}{w_0}\quad \text{or}\quad \left(\frac{G_i}{G_0}\right).$$

Apart from the method of similarity based on the reduction of the system of equations to dimensionless form, the method of dimensional analysis is also used for determining the system of criteria required for generalizing the characteristics of electric arcs. In this case, the number of dimensionless arguments is determined from Buckingham's pi theorem.

In [6], in which dimensional analysis was employed, the quantities important for the process were assumed to be: the masses of the molecules of the starting gas, the masses of all the particles formed as a result of thermal dissociation and ionization, the dissociation and ionization potentials, the excitation potentials, the collision cross sections, the electronic charge, the specific heat of the starting gas, the magnetic permeability and Boltzmann's constant, the velocity of the gas admitted to the discharge chamber, the current, the temperatures of the chamber walls and the gas flow, the pressure, the magnetic field strengths, the electrode fall potentials, the cathode work function, and also the geometry of the discharge chamber.

When only two characteristic dimensions were taken, it was found that the process is determined by m + n + q + + 23 primary independent quantities, where m is the number of potentials taken into account, n the number of particles of different species, and q the number of collision cross sections taken into account. In the case of four fundamental units m + n + q + 19 criteria are obtained. Thus, the total number of dimensionless arguments runs into the tens.

However, when dimensional analysis is used, it is necessary to employ unusual skill and considerable intuition in selecting the quantities important for the process and determining the form of the dimensionless complexes. This is quite a difficult problem even when the number of fundamental quantities is small and becomes incredibly more complicated as it increases.

II. GENERALIZATION OF CURRENT-VOLTAGE CHARACTERISTICS IN DIMENSIONAL COMPLEXES

System of equations (1)–(6) was obtained with a number of simplifying assumptions. Therefore, the dimensionless complexes derived from it are only part of the total system of criteria and their use permits only an approximate reflection of the similarity of the processes. However, even the system of complexes obtained is quite clumsy. Yet if we take into account the other disregarded processes, the number of dimensionless arguments must increase considerably.

To a considerable extent, the number of effects that play an important role depends on the number and range of variation of the primary quantities. The number of necessary criteria varies correspondingly. However, increasing the number of generalized arguments sharply complicates the experimental determination of the laws involved and renders difficult the choice of a suitable approximating expression and the application of the formulas obtained. It may even prove that the difficulties of approximation do not permit an improvement in the accuracy of the formulas with increase in the number of variables, so that a great deal of experimentation is done in vain.

The narrower the range of variation of the primary quantities, the easier it is to neglect a number of less important processes. Accordingly, from this standpoint, it is desirable to restrict (within reasonable limits) the region of variation of the characteristic quantities. It is then possible to neglect a number of criteria. For example, in the absence of external magnetic fields it is possible to disregard the complex Π_2, and if the current is small, the complex Π_3 as well. At small pressure gradients in the arc column the pressure forces may prove to be incommensurably small as compared with the inertia forces, so that the criterion Π_1 can likewise be discarded. If the variables Π_2, Π_3 are also neglected, the force interactions are no longer taken into account at all. In exactly the same way, in flow-through arcs it is possible to neglect the effect of the criterion Π_7, and in stabilized arcs at small flow rates the criteria Π_5, Π_6. However, even for a small region of variation of the primary quantities, the choice of suitable criteria, starting from physical considerations, is a rather difficult matter. At the same time, expansion of the range of the investigated parameters and experimental determination of the important processes involves considerable time and expense. Clearly, this is the reason for the slow progress in generalizing the characteristics of plasma generators obtained experimentally.

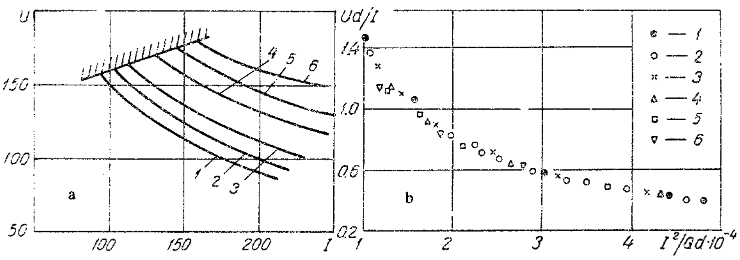

Fig. 1. Current-voltage characteristics (a—ungeneralized b—generalized) of a linear plasma generator with vortex air stabilization (d = 1 cm; U, V; I, A): 1) G = 1.01 g/sec; 2) 1.15; 3) 1.26; 4) 1.4–1.6; 5) 1.183; 6) 2.05.

The first successful attempt to generalize the characteristics of an electric-arc heater was made in [1].

Starting from the definition of a plasma generator as a gas heater, by dimensional analysis the authors obtained the criterion Π_5 and generalized the current-voltage characteristics of heaters with vortex air and nitrogen stabilization in the form $\Pi_8 = f(\Pi_5)$.

An example of the generalization of data from [2] is presented in Fig. 1. Here, as the generalized function we have taken the complex $\Pi_8' = UL\sigma_0/I$. Clearly, the generalization is quite satisfactory.

It should be noted that the generalizations were carried out for each of the gases separately in dimensional complexes corresponding to the dimensionless criteria, but not containing the physical properties. This created difficulties in determining the scales of reference for the electric arc. If they are taken with respect to the temperature of the cold gas and the pressure in the discharge chamber, which would be quite natural, starting from the known boundary conditions, the electrical conductivity is almost equal to zero. However, the temperature of the arc column, at which the process takes place, is not known in advance, since it itself depends on other characteristic quantities. However, if we take the scales of reference at some sufficiently high temperature, where the electrical conductivity has a suitable value, it may turn out that for different gases this temperature corresponds to different points of the field and similarity does not exist.

The authors of [1] used the fact that the scales of reference are constant for a given gas and carried out the generalization in dimensional complexes, without determining the specific value of σ_0 and h_0. Dimensional complexes were also used in subsequently published work.

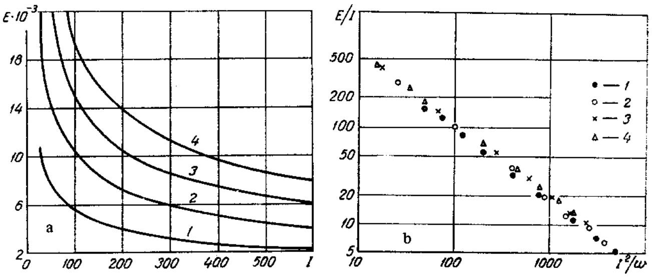

Fig. 2. Current-voltage characteristics of an electric arc moving in air (a and b see Fig. 1). (E, $V \cdot m^{-1}$; I, A; w, $m \cdot sec^{-1}$; 1) w = 200 $m \cdot sec^{-1}$; 2) 150; 3) 100; 4) 50.

Another feature of [1] is the rather narrow range of variation of the primary quantities. For example, the data of Fig. 1 relate to only one value of the electrode diameter. In subsequent studies this limitation was gradually overcome. In [3] electric-arc devices with sharply different stabilization conditions were considered and the characteristics of each were generalized by means of a single argument. In particular, it was shown that the current-voltage characteristics of water-stabilized arc, obtained in [4], are satisfactorily generalized in the dimensional complexes $Ed^2/I - I/d$, which corresponds to the dimensionless criterial relation $\Pi_8 = f(\Pi_7)$. This was to be expected, since almost all the energy is extracted from the stabilized arc in the radial direction by heat conduction, while not much gas flow energy is removed through the ends of the diaphragm. In the case of a transverse flow over the arc, on the other hand, the current-voltage characteristics are satisfactorily generalized in the form $\Pi_8 = f(\Pi_5'')$, but in this case, by condition, the diameter of the arc column is not given, since it depends on the primary quantities (current, gas velocity). Therefore, the data of [5] were generalized in the dimensional complexes $E/I - I^2/w$. Figure 2 shows how the characteristics of an arc in a transverse flow are generalized. It is clear from the figure that in logarithmic coordinates the generalized current-voltage characteristic has the form of a straight line. Therefore it is conveniently approximated by a power function. The expression corresponding to Fig. 2 has the form

$$E/I = 3550\,(I^2/w)^{-0.76}. \tag{7}$$

The units and ranges of variation of the quantities are given in the figure.

It is not possible, however, to obtain such a simple relation for the data presented in Fig. 3, since the characteristic is not a straight line. It consists, as it were, of two segments meeting at the point $I/d \simeq 2 \cdot 10^4$. The left part corresponds to the descending branches of the current-voltage characteristics shown in Fig. 3a and is steeper than the right part, corresponding to the ascending branches of the EI-characteristics.

The following formula was obtained for the left segment:

$$\frac{Ed^2}{I} = 400\left(\frac{I}{d}\right)^{-\frac{1}{3}}. \tag{8}$$

The units and ranges of variation of the quantities are given in the figure. For the ascending branch a power-law approximation is not convenient. But selecting a sufficiently simple approximation of another type is very difficult. Accordingly, greater accuracy can be achieved by using the graph.

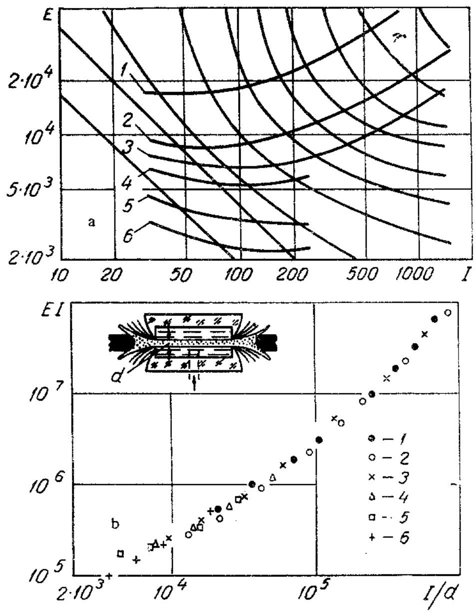

Fig. 3. Current-voltage characteristics of water-stabilized arc (a and b see Fig. 1) (E, V · m^{-1}; I, A; d, m): 1) d = 0.14 cm; 2) 0.23; 3) 0.33; 4) 0.40; 5) 0.70; 6) 1.14.

In [6] it was shown that the relation $\Pi_8 = f(\Pi_7)$ is also suitable for generalizing arcs stabilized by a cooled segmented metal diaphragm, if the gas flow rates are not large. The characteristics of a stabilized arc burning in an argon medium [7] have been generalized in the form $Ed^2/I = f(I/d)$. In [6] it was also shown that there is an interrelation between the criteria Π_3 and Π_4 represented in the form $L\rho_0^{0.5}w/I\mu_0^{0.5}$ and I/LH, respectively. The data of [5] were used for the generalization.

Other authors have also generalized the current-voltage characteristics of blown arcs in the form $\Pi_8 = f(\Pi_5'')$. In [13] plasma generators with self-regulating and fixed arc lengths were investigated. At I = 40–180 A, G = 3.3–6.9 g/sec, d = 1.0 cm, and l = 0.3–7 cm and constant ratios of the air flow rates through the gaps it was found possible to disregard all the generalized arguments other than Π_5. The data presented in the figure yield the following formulas:

$$\frac{Ud}{I} = 6.8 \cdot 10^4 \left(\frac{I^2}{Gd}\right)^{-0.76} \tag{9}$$

for a plasma generator without an interelectrode insert and

$$\frac{Ud}{I} = 3.7 \cdot 10^3 \left(\frac{I^2}{Gd}\right)^{-0.595} \tag{10}$$

for a heater with an insert. The units in these formulas are [m], [kg/sec], [A], and [V].

When an insert is used, the characteristics fall less steeply and the arc voltage is higher.

In [14] the arc characteristics were calculated analytically on the assumption that the gas is heated as a result of turbulent mixing. The analysis showed that the number Π_8 depends on Π_5. The calculations are in good agreement with experiments using air, nitrogen, hydrogen, and helium. In [15] a similar calculation was made for an arc burning in lithium vapor at pressures of 5–1000 N/m^2. In this case experiment revealed a certain stratification of the $\Pi_8 = f(\Pi_5'')$

curves depending on the gas flow rate. Accordingly, in the presence of a broad range of variation of the characteristic quantities it is necessary to take other criteria into account as well.

In generalizing the characteristics of a plasma generator with a self-regulating arc length, the authors of [6] proposed taking into account, in addition to the energy relations of the force of friction, the phenomenon of electrical breakdown of the cold layer adjacent to the electrode. For this purpose, it was proposed to introduce into the generalized formulas the Re and Kn numbers, which in dimensional form are represented as G/d and 1/Pd, respectively.

For a plasma generator with a fixed arc length the criterion d/a has been introduced, where a is the length of the stabilizing diaphragm in which the arc burns. If there are several points of gas admission, then the parametric criteria G_i/G are also introduced [26].

For simplicity, however, the authors of [8] also used individual combinations of the above-mentioned criteria. In particular, for generalizing blown arcs in dimensional form instead of the complex I^2/Gd they propose the use of I/d or I/G.

As shown above, the complex I/d corresponds to the criterion $1/\Pi_7$. It is convenient for generalizing the characteristics of stabilized arcs at very small gas flow rates and for swept arcs, if the energy is removed from the column by heat conduction. The authors of [8] used this complex for large flow rates, but treated it not as a dimensional part of the criterion $1/\Pi_7$, but as part of the combination of dimensionless quantities $(\mathrm{Re}/\Pi_5)^{0.5}$. Similarly, the combination $(1/\mathrm{Re}\Pi_5)^{0.5}$ gives in dimensional form the complex I/G. In this connection, it should be noted that there is no difference between the criteria Pe and Re in dimensional form (both give the complex G/d). But the ratio $\mathrm{Pe}/\Pi_5 = 1/\Pi_7$ has a perfectly definite physical significance. For swept arcs its use is completely justified, whereas the ratio Re/Π_5 does not characterize any physical model. In exactly the same way, the product $\mathrm{Re}\Pi_5$ (or, more correctly, $\mathrm{Pe}\Pi_5$) is convenient to use if the degree of influence is the same.

When the primary quantities are varied in the range I = 30–200 A, d = 0.6–3.5 cm, G_{air} = 4.1–2.4 g/sec, and P = 10 N/cm^2, in the case of reverse polarity (the outer electrode is the cathode) we obtain the relation

$$\frac{Ud}{I} = 1.3 \cdot 10^4 \left(\frac{G}{d}\right)^{0.3} (Pd)^{0.04} \left(\frac{I}{d}\right)^{0.245\lg(I/d)-2.312} \mathrm{V\cdot cm/A} \tag{11}$$

which, however, the authors present not in the given form, but in the form of the usual empirical equation with isolation of the complex I/d:

$$U = 1.3 \cdot 10^4 G^{0.3} d^{-0.26} P^{0.04} \left(\frac{I}{d}\right)^{0.245\lg(I/d)-1.312}. \tag{11'}$$

Generalizations of the current-voltage characteristics in the form $\frac{Ud}{I} = f\left(\frac{I}{d};\ Pd;\ \frac{G}{d}\right)$ gave somewhat better results than in the form $\frac{Ud}{I} = f\left(\frac{I^2}{Gd};\ \frac{G}{d};\ Pd\right)$. However, since no new criteria have been introduced, and the same three criteria (Π_5, Re, Kn), have only been combined, the improvement in the results is evidently attributable to the superior choice of an approximating expression, since the analysis in the form $\Pi_8 = f(\Pi_5, \mathrm{Re}, \mathrm{Kn})$ was carried out by means of a power-law approximation.

The insufficiency of the criterion Π_5 has also been demonstrated in a number of other investigations, whose authors, however, proceed by introducing new dimensionless arguments. The authors of [9–11] assume that, apart from the criterion Π_5, it is necessary to take into account the relation between the diameter of the arc column and the electrode diameter. However, since the diameter of the arc column is not known, they assume that the cross section of the arc column is proportional to the power released and inversely proportional to the energy removed from the surface of the arc by convective heat transfer. Actually (if we assume not R_h = const, and p = const), this corresponds to the swept model, in which the arc is treated as a solid rod, whose electrical resistance, heat-transfer coefficient and temperature drop do not depend on the arc burning conditions:

$$4I^2 l/\pi\sigma (d')^2 = \pi d' l \alpha \Delta t. \tag{12}$$

Hence, with the above assumptions, we obtain $I^2/(d')^3$ = const. Substituting the value of d' in the ratio d/d' gives the dimensional complex $I^{\frac{2}{3}}/d$, which has also been used for generalizing the characteristics of swept arcs.

Analysis of the experimental data of [12] led to the relation

$$\frac{Ud}{I} = 3900\left(\frac{I^2}{Gd}\right)^{-0.33}\left(\frac{I^{\frac{2}{3}}}{d}\right)^{-1.0} \text{V}\cdot\text{cm/A} \tag{13}$$

for a plasma generator with a rod cathode at I = 5–12 A, G_{hydrogen} = 0.48–1.63 g/sec, d = 1–3 cm, and

$$\frac{Ud}{I} = 4500\left(\frac{I^2}{Gd}\right)^{-0.33}\left(\frac{I^{\frac{2}{3}}}{d}\right)^{-1.0} \text{V}\cdot\text{cm/A} \tag{14}$$

for a plasma generator with a tubular cathode.

The above method of generalization calls for some comment. First of all, it is not possible to use as a generalized argument the ratio of the arc and electrode diameters d/d', since the diameter of the arc column is not known in advance and depends on the arc burning conditions. Consequently, d/d' is not a criterion, but simply a dimensionless number, which can be used as a generalized function, but not as an argument. The transformations make it possible to obtain a complex in which all the variables are known (I,d), but, as before, this complex is only part of a dimensionless number, and not a criterion, since it follows from (12) that in dimensionless form the number must contain such quantities as α and Δt, which are not physical properties of the gas and depend on the arc burning conditions.

Since the authors eliminate d' by means of relation (12), they in fact use two energy complexes I^2/Gd and $I^{2/3}/d$, reflecting two directly opposite arc models—flow-through and swept, i.e., the actual process is in fact "decomposed" into the two limiting cases. The assumptions regarding the constancy of the electrical conductivity, temperature drop and heat transfer coefficient are very rough, since as the current changes so do the velocity of the gas sweeping over the arc, the temperature and the electrical conductivity.

In [16], generalizations of the characteristics of a plasma generator with a cooled segmented diaphragm operating, however, at relatively large gas flow rates, showed the expediency of using the criteria Π_5, Re, and l/d. The following formula in dimensional-dimensionless complexes was obtained for argon:

$$\frac{Ud}{I} = 58\left(\frac{I^2}{Gd}\right)^{-0.38}\text{Re}^{-0.27}\left(\frac{l}{d}\right)^{0.65}. \tag{15}$$

The Reynolds number was calculated from the viscosity for the cold gas. The regions of variation of the parameters where: G = 1–6 g/sec, I = 100–1000 A, l = 2–18, cm, d = 1.8 cm.

It should be noted that in the given case there is no special need to write the Re number in dimensionless form, since Π_8 and Π_5'' are dimensional, the more so in that the Peclet rather than the Reynolds number may be involved.

Investigations of a plasma generator with a self-regulating arc length operating at an underpressure in the discharge chamber (P = 32–760 mm Hg) showed [17] that it is necessary to use the Re number. For the regions P = = 32–755 mm Hg, I = 16–160 A, G = 0.432–2.5 g/sec, d = 0.8 cm the following expression was obtained in the above-mentioned units:

$$\frac{Ud}{I} = 1\cdot 10^4\left(\frac{I^2}{Gd}\right)^{-\frac{3}{4}}\text{Re}^{-0.5}. \tag{16}$$

In this formula, the Reynolds number is calculated from the mass-averaged parameters of the heated gas, which is convenient when the temperature at the plasma generator outlet is given. Otherwise this expression is difficult to employ.

As may be seen from Fig. 4, the generalizations based on (16) are good. However, it should be kept in mind that here the criterion Π_5 and the number Π_8 are represented in dimensional form and the physical properties σ_0 and h_0 remain constant, whereas μ_0 in the Reynolds number varies together with the temperature of the heated gas. If, however, the expression is made completely dimensionless or completely dimensional, the generalized curve becomes stratified. Consequently, the generalization of the current-voltage characteristics of a plasma generator with the scale values referred to the mean-mass parameters is evidently not a successful solution of the problem.

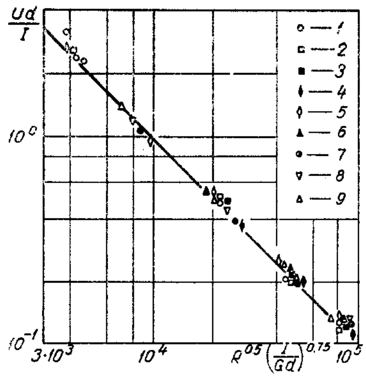

Fig. 4. Generalized current-voltage characteristics of a vacuum plasma generator with vortex gas stabilization (P = 32–755 mm Hg; I, A; U, V; G, g·sec^{-1}; d, cm): 1) P = 32–47 mm Hg; G = 0.432–0.446 g·sec^{-1}; 2) 56 and 0.44–0.449; 3) 95 and 0.439–0.444; 4) 755 and 0.436; 5) 50–82 and 0.873–0.883; 6) 101–105 and 0.878–0.880; 7) 750 and 0.878–0.885; 8) 775 and 1.460–1.480; 9) 755–766 and 2.47–2.50.

Moreover, generalization using scale values referred to the gas parameters at the outlet is also inconvenient when the gas is heated to low temperatures: the electrical conductivity may then be equal to zero. Furthermore, the outlet parameters are affected by the dimensions and configuration of the outlet part of the plasma generator, whose variation has almost no influence on the arc characteristics.

In coaxial plasma generators with magnetic rotation of the arc it becomes necessary to introduce the criterion Π_2. This can be seen from [18], where the characteristics of a coaxial plasma generator are generalized in the form $\Pi_8 = f(\Pi_5, \Pi_2^{\prime}, \delta/G)$, where δ is the interelectrode gap. It should be noted, however, that for plasma generators it is usually the gas flow rate and not the velocity that is given; accordingly, it would be more convenient to use not the criterion Π_2, but $\Pi_2^{\prime\prime}$.

In generalizing the current-voltage characteristics of a plasma generator with a self-regulating arc length additional difficulties are introduced by the electrical breakdown of the layer of cold gas adjacent to the electrode. In the first attempts at generalization [1] this effect was not taken into account. It was assumed that the thickness of the layer and the breakdown voltage depend to a considerable extent on the heating of the gas on the previous section of the electrode and, consequently, are also determined by the criterion Π_5. In [6] the Knudsen number was introduced to allow for the breakdown effect. The author of [19] went even further in this direction. Starting from the existence of a relationship between breakdown voltage and gas temperature, he proposed using only one criterion, characterizing the breakdown effect. The generalized characteristics were represented in the form:

$$\frac{U}{U_i} = f\left(\frac{A_0 p d c_p G T_0}{I U_i}\right). \tag{17}$$

For purposes of a power-law approximation in dimensional complexes the following expression was used for the generalizations:

$$U = c\,(Gdp/I)^{\alpha}. \tag{18}$$

For ordinary polarity (the outlet electrode is the anode) $c = 1140$, $\alpha = 1.41$, for reverse polarity $c = 1230$, $\alpha = 0.33$, and for a two-sided plasma generator $c = 1660$, $\alpha = 0.33$. The experiments were conducted at I = 10–250 A, G_{air} = 0.5–30 g/sec, d = 0.5–2 cm. The units of the quantities in formula (18) are G, g/sec; d, cm; P, bar; I, A; U, V.

In the investigated range of variation of the primary quantities the above method gave fairly good results—the error did not exceed ±28% (Fig. 5). However, the representation of the basic process (the heating of the gas) in veiled form seriously reduces the value of the method. Moreover, the choice of ionization potential as a scale value assumes that the effect of the breakdown processes is more important than the heating of the gas. This view is not consistent with the general trend of research.

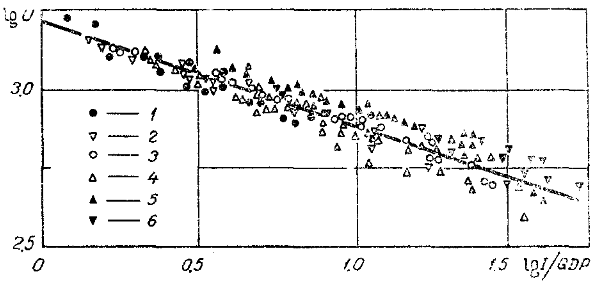

Fig. 5. Generalized current-voltage characteristic of a dc plasma generator with vortex air stabilization (U, V; G, g/sec; d, cm; P, bar; I, A): 1) d = 2 cm; 2) 1.5; 3) 1.2; 4) 1.0; 5) 0.8; 6) 0.5.

Other investigations [20–22] show that many factors affect the arc length in plasma generators with vortex gas stabilization: the ratio of arc and electrode diameters, the turbulence of the flow, the radiation of the arc, external magnetic fields, etc. Hence, depending on the operating regime of the device the voltage may both increase and decrease with increase in pressure [21]. At the same time, if all the other dimensional complexes, apart from Pd, are equal, the generalized function depends more strongly on d than on P. Therefore the Knudsen number is clearly insufficient for taking into account the complex of processes affecting the arc length [20]. However, no additional criteria have been proposed and the authors of [20–23], together with the complexes I^2/Gd; G/d, introduced into the formulas the electrode diameter d, which, in their opinion, represents the missing unknown criteria. In particular, in [20] the following formula was proposed for geometrically dissimilar plasma generators with vortex air stabilization:

$$\frac{Ud}{I} = 1.04 \cdot 10^5 \left(\frac{I^2}{Gd}\right)^{-0.69} \left(\frac{G}{d}\right)^{-0.25} d^{0.36}, \tag{19}$$

This was obtained by generalizing the experimental data in the range of variation of the primary quantities: I = 100–800 A, G = (2–6) $\cdot 10^{-3}$ kg/sec, d = (1–4) $\cdot 10^{-2}$ m. However, such an expression is valid only when the diameter of the inner electrode exceeds the diameter of the the outer cylindrical electrode by a factor of not less than 2 (D/d > 2). If, however, D/d < 2, this parametric criterion must also be taken into account.

The difficulties of generalizing the experimental data when the characteristic quantities vary over a sufficiently broad range have compelled many investigators to abandon the theory of similarity altogether. A number have turned to ordinary empirical formulas or complexes that are not part of dimensionless criteria [8, 9, 24, 25]. For example, the authors of [24] propose the formula

$$U = \frac{5620}{10^{0.03M}} \left(\frac{G^2}{Id}\right)^{1.3} \text{в}, \tag{20}$$

obtained for I = 200–1200 A; $G_{nitrogen}$ = 2–10 g/sec; G_{air} = 1.01–13.25 g/sec; G_{oxygen} = 4–13.8 g/sec; d_{an} = 1–2 cm.

The left-hand side of this equation is simply the arc voltage, and not part of a dimensionless complex, while the complex on the right-hand side is not part of any dimensionless criterion.

The usual empirical formula for plasma generators with vortex nitrogen stabilization $U = f(I, G)$ is presented, for example, in [25].

The return to empirical formulas, however, cannot satisfy investigators convinced of the fruitfulness of the methods of approximate similarity. Efforts to find the basic processes affecting the characteristics of arc heaters and the corresponding criteria continue. Of considerable interest in this connection is reference [27], whose authors propose that, apart from shunting, the length of the arc is affected by a number of other processes leading to the deceleration of the velocity of the radial segment in the gas stream. In this case, we get a transverse flow over the part of the arc near the electrode accompanied by a pressure drop [22]. In this connection, the authors of [27] propose using the Euler number for taking into account the force interactions. Since the scale of reference was assigned in the form of the known absolute value of the pressure, in fact, the criterion Π_1, which is a generalized argument, was used.

For a plasma generator with air stabilization at $I = 100–850$ A; $G = (6–8) \cdot 10^{-3}$ kg/sec; $D_c = (2.0–3.5) \cdot 10^{-2}$ m; $d_{an} = (0.8–2.0) \cdot 10^{-2}$ m (the cathode is the inner cup-shaped electrode, the anode the outer cylindrical electrode); $P = (1.06–10.1) \cdot 10^5$ N/m^2 the following formula was obtained:

$$\frac{UD}{I} = 5.62 \cdot 10^3 \left(\frac{I^2}{GD}\right)^{-0.66} \left(\frac{PD^4}{G^2}\right)^{0.14} \left(\frac{G}{D}\right)^{0.06}. \tag{21}$$

The scatter of the experimental points did not exceed ±20%, or, disregarding the complex G/D, ±25%. This study completely neglected the phenomena of breakdown of the electric layer of gas, whose existence has been proved by a number of investigators [28–30]. Consequently, the authors of [27], while retaining the original viewpoint regarding the dependence of shunting processes on other factors (heating processes and interacting forces), were able with a small number of criteria to generalize the experimental data obtained over a quite broad range of variation of the primary quantities. The use of the criterion PD^4/G^2 made it possible to eliminate the parametric criterion D/d introduced in [20] and free the generalized formulas from the electrode diameter d, which does not enter into any of the complexes. At the same time, the relation between the exponents of P and D in the criterion Π_1 is in better agreement with the results of experiments than in the Knudsen number.

However, it should not be assumed that the authors of [27] completely succeeded in proving the primary influence of the force interactions and the dependence of shunting on heating. This is clear if only from the fact that the accuracy of the generalizations of the same experimental data in dimensional complexes corresponding to the criteria Π_5, Re, and Kn is approximately the same as in the complexes corresponding to the criteria Π_1^r, Re, Π_5^r. Generalizations in dimensionless form would apparently clarify the situation, since the physical properties in the criteria Π_1^r and Kn are essentially different.

III. GENERALIZATION OF CURRENT-VOLTAGE CHARACTERISTICS IN DIMENSIONLESS CRITERIA

As already pointed out in the first section, the generalization of the current-voltage characteristics in dimensionless criteria is a complicated problem owing to the impossibility of selecting a scale value of the electrical conductivity under the boundary conditions.

So far only one method of selecting the conductivity scale has been proposed [31]. The idea consists in typing the point at which the physical properties are equal to the scale values to some definite part of the arc column. It is not possible to use the maximum temperature along the axis of the column, since this temperature itself depends on the arc burning conditions. It is possible to be more definite about the temperature at the boundaries of the arc, since the beginning of the sharp increase in electrical conductivity is known for each gas. It is particularly easy to define the point of completion of ionization, beyond which the sharp growth of electrical conducitivity as a function of temperature (or enthalpy) is replaced by a flatter section of the curve. For heavy-current arcs, whose characteristics are determined basically by the inner part of the column, where ionization is essentially complete, this point is actually located on the boundary of the arc column.

In [31] the $\sigma = f(h)$ curve in logarithmic coordinates were approximated by two straight lines: an initial interval of rapid growth of electrical conductivity—vertical line, and a second sloping interval. The jog was taken as the scale point. In this way (Fig. 6) the values of σ_0 and h_0 for certain frequently used gases were determined at atmospheric pressure. These values of σ_0 and h_0, together with the exponent of the function $\sigma = \sigma_0(h/h_0)^n$, approximating the sloping part of the curve, are presented in the table. The values of σ_0, h_0, and n depend slightly on pressure.

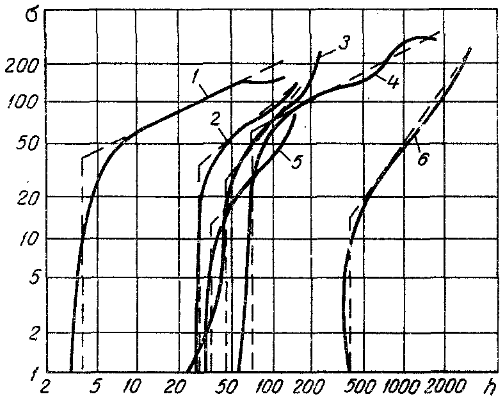

Fig. 6. Approximation of the relation $\sigma = f(h)$ at atmospheric pressure (σ, $ohm^{-1} \cdot cm^{-1}$; h, kJ/g): 1) argon; 2) oxygen; 3) nitrogen; 4) helium; 5) lithium; 6) hydrogen.

In [31] this method was used to generalize the characteristics of a dc plasma generator with vortex gas stabilization for argon (4–12 g/sec), helium (1–4 g/sec), oxygen (4–8 g/sec), and nitrogen (2–6 g/sec). In the experiments the current varied from 200 to 1200 A, the diameters of the anode and cathode remained unchanged and equal to 2 cm.

Correct to 30%, the results are approximated by the formula

$$\frac{Ud\sigma_0}{I} = 3.39\left(\frac{I^2}{Gd\sigma_0 h_0}\right)^{-0,62}. \tag{22}$$

However, as may be seen from Fig. 7, the slope of the generalized characteristic differs considerably from that of the characteristics of other gases, therefore, in their improved formula, the authors propose taking this into account and approximating the experimental data with the expression

$$\frac{Ud\sigma_0}{I} = an^l\left(\frac{E_i}{kT}\right)^m\left(\frac{I^2}{Gd\sigma_0 h_0}\right)^{-b}, \tag{23}$$

where $l = -0.84$; $m = -1.0$; $a = 1620$; $b = cn^p (E_i/kT)^q$. Here, $c = 0.05$, $p = 0.34$ and $q = 0.4$.

Values of the Coefficients in the Power-Law Approximation of the Relation Between Electrical Conductivity and Enthalpy $h < h_0$ $\sigma = 0$, at $h > h_0$ $\sigma = \sigma_0(h/h_0)^n$)

Stabilizing Medium	$ohm^{-1} \cdot cm^{-1}$	kJg^{-1}	n
Hydrogen	14	400	1.38
Helium	63	80	0.514
Lithium	13	35	1.12
Nitrogen	27	46	1.21
Air	29	44	1.19
Oxygen	34	29	0.81
Argon	40	4	0.48

As may be seen from this expression, apart from the criterion Π_5'', it was necessary to use the exponent n and the criterion E_i/kT, which reflect the physical properties of the gas. The use of the latter criterion cannot be considered successful, since it is not obtained directly from the simplified system of equations (1)–(6). Apparently, the authors were also somewhat hasty in deriving the dependence of the slope of the characteristic on the nature of the gas.

The method of selecting the scales of reference proposed in [31] was employed in [32,33] for generalizing the current-voltage characteristics of geometrically dissimilar dc plasma generators with vortex stabilization of the arc with argon, nitrogen, air, helium, oxygen, and hydrogen. In a development of [27] the generalizations were carried out in the form $\Pi_8 = f(\Pi_5'', \Pi_1'', n)$. Generalization of the experimental data at I = 13–860 A, G_{argon} = 1–12 g/sec, G_{helium} = 0.25–4 g/sec, $G_{hydrogen}$ = 0.5–3.0 g/sec, G_{air} = 0.35–18 g/sec, $G_{nitrogen}$ = 2–10 g/sec, G_{oxygen} = 8.5–30 g/sec, D = 1–4 cm (inner cup cathode) and d = 0.8–4 cm (outer cylindrical anode) led to the expression

$$\frac{UD\sigma_0}{I} = 0.429\left(\frac{I^2}{GD\sigma_0 h_0}\right)^{-0.613}\left(\sqrt{\frac{\rho_0}{p_0}}\,\frac{PD^2}{G}\right)^{0.254} n^{-0.4}. \tag{24}$$

The accuracy of the formula is rather low, the scatter of the experimental points reaching ±100% (Fig. 7). However, this is not its principal disadvantage. If we confine ourselves to the investigation of one particular geometric configuration of the discharge chamber, narrow the region of variation of the parameters and select the most suitable approximating expressions, the error can be reduced to an acceptable level. The possibility of using the criterion Π_1 for generalization purposes is doubtful per se. In the heater electrode the pressure drop is due mainly to the acceleration of the gas in the region of contraction of the channel near the radial section of the arc and the acceleration of the flow as it is heated along the electrodes. However, flow over the radial section of the column is possible only if it is restrained by certain forces. In this case, it is convenient to introduce into the generalized formulas criteria reflecting these primary processes. However, the acceleration of the gas during heating is possible in principle even if the pressure does not change. And in fact this is what happens—the criterion is suitable for analyzing the experimental data at very small pressure drops in the discharge chamber, which contradicts its essential nature. Apparently, it is somehow related with other criteria reflecting the process of acceleration of the gas during heating.

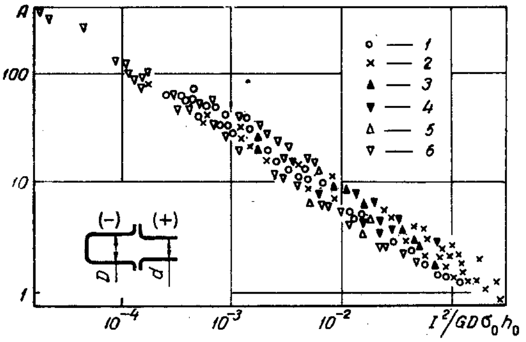

Fig. 7. Generalized current-voltage characteristic of a dc arc heater with vortex gas stabilization: 1) air; 2) argon; 3) nitrogen; 4) hydrogen; 5) helium; 6) oxygen.

$$A = \frac{UD\sigma_0}{I} \Big/ \left(\sqrt{\frac{\rho_0}{P_0}}\,\frac{PD^2}{G}\right)^{0.245} n^{-0.4}$$

An attempt to eliminate this shortcoming by replacing the criterion Π_1' by the complex Π_6', reflecting the process of acceleration of the gas due to Joule heating, has been made by Zhidovich and Yas'ko. At the same time, to take into account the temperature dependence of the plasma properties, instead of the exponent n they introduce the enthalpy factor h_0/h_{in}. This made it possible to reduce the scatter of the experimental points to ±40%, without restricting the range of variation of the primary quantities. An example of the generalized characteristic is presented in Fig. 8. Here one can distinguish two regions of different steepness. The left-hand region corresponds to the descending ($U \sim I^{-0.32}$), and the right-hand region to the ascending ($U \sim I^{0.04}$) branch of the current-voltage characteristic. Of course, in the latter case the increase would be greater if the length of the arc did not grow shorter as the current increased.

The decending branch can be approximated by the expression

$$\frac{UD\sigma_0}{I} = 4\left(\frac{I^2}{GD\sigma_0 h_0}\right)^{-0.81}\left(\frac{I^2\rho^2D^3}{\sigma_0 G^3}\right)^{0.15}\left(\frac{h_0}{h_{\text{in}}}\right)^{-0.33}. \tag{25}$$

Correspondingly, for the ascending branch

$$\frac{UD\sigma_0}{I} = 8.3\left(\frac{I^2}{GD\sigma_0 h_0}\right)^{-0.63}\left(\frac{I^2\rho^2D^3}{\sigma_0 G^3}\right)^{0.15}\left(\frac{h_0}{h_{\text{in}}}\right)^{-0.33}. \tag{26}$$

The accuracy of these formulas is already sufficient to permit their use in engineering calculations. However, the error could be reduced considerably more by selecting more suitable approximating expressions. Among other things, this is indicated by the fact that the exponent of the criterion Π_6' varies from 0.1 to 0.2 in the investigated range (more than four orders). Its mean value of 0.15 was taken in the approximation. The degree of influence of the enthalpy factor also varies.

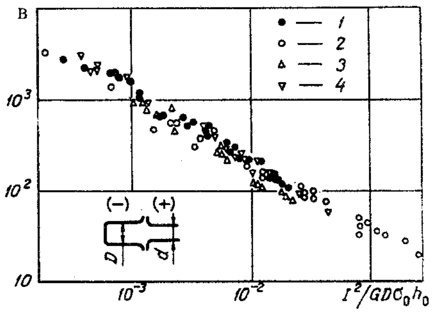

Fig. 8. Generalized current-voltage characteristics of electric-arc heater with vortex gas stabilization: 1) air; 2) argon; 3) hydrogen; 4) helium.

$$B = \frac{UD\sigma_0}{I} \Big/ \left(\frac{I^2\rho^2D^3}{\sigma_0 G^3}\right)^{0.15}\left(\frac{h_0}{h_{\text{in}}}\right)^{-0.33}$$

It should also be noted that instead of one of the complexes Π_5'' and Π_6' it is better to use their ratio. This considerably facilitates the correlation of the experimental data, since during the experiments the dimensions of the discharge chamber and the gas flow rate are kept fixed, while the current is varied. It is therefore desirable that the current should enter into only one of the criteria. With this substitution instead of (25), (26) we obtain

$$\frac{UD\sigma_0}{I} = 4\left(\frac{I^2}{GD\sigma_0 h_0}\right)^{-0.66}\left(\frac{G}{\rho\sqrt{h_0}D^2}\right)^{-0.3}\left(\frac{h_0}{h_{\text{вх}}}\right)^{-0.33}, \tag{25'}$$

$$\frac{Ud\sigma_0}{I} = 8.3\left(\frac{I^2}{GD\sigma_0 h_0}\right)^{-0.48}\left(\frac{G}{\rho\sqrt{h_0}D^2}\right)^{-0.3}\left(\frac{h_0}{h_{\text{вх}}}\right)^{-0.33}. \tag{26'}$$

Since $\rho = \rho_0 P/P_0$, in dimensional form, when ρ_0, h_0, and P_0 are constant, the criteria Π_1'' and Π_{12}'' completely coincide. For a certain given gas at constant heater inlet temperature the h_0/h_{in} are also fixed, and Eqs. (25') and (24) coincide. Hence it is clear why the criterion Π_1'' is suitable for generalizing the characteristics even in the absence of pressure drops. It does, in fact, characterize the processes of acceleration due to heating of the gas. In dimensionless form, with account for the equation of state, the ratio Π_{12}/Π_1'' is equal to c_{p0}/R. This leads to an important difference between monotonic and polyatomic gases. However, the use of the exponent n in Eq. (24) smoothes over this difference somewhat.

Expressions (25') and (26') relate to heaters with self-regulating arc length. Therefore, it is not clear whether the acceleration of the gas affects both the arc column in the longitudinal flow and the length of the arc in the transverse flow over the radial section or whether the effect extends only to one of these sections. To answer this question, it is necessary to carry out special experiments. It is already clear that this involves considerable difficulties, since as shown in [34, 35], the strength of the electric field varies along the length of the arc column and the laws of variation of the current have not been established. Nonetheless, by means of a generalization of the current-voltage characteristics of heaters with a fixed arc length it is possible to some extent to determine the effects of the acceleration of the gas on the characteristics of the column.

The authors of [36] have investigated the characteristics of a segmented-electrode heater at $I = 300-3000$ A, $d_c = 1.0-2.0$ cm. The arc had a fixed length $l_a = 7.0-15.4$ cm and was air-stabilized, $G = 0.4-15.0$ g/sec, $P = 0.005-0.3$ kg/cm^2. Generalization of the experimental data gave the following expression:

$$\frac{Ud_{\kappa}^2}{lI} = 5\left(\frac{I^2}{Gd_{\kappa}}\right)^{-0.4}\left(\frac{G}{d_{\kappa}}\right)^{-\frac{1}{3}} d_{\kappa}^{\frac{1}{3}}. \tag{27}$$

Obviously, by combining the last two numbers, we can obtain the dimensional complex G/d_c^2 corresponding to the criterion:

$$\frac{Ud_{\kappa}^2}{l_a I} = 5\left(\frac{I^2}{Gd_{\kappa}}\right)^{-0.4}\left(\frac{G}{d_{\kappa}^2}\right)^{-0.33}. \tag{27'}$$

This complex enters into the generalized formula to approximately the same power as in expressions (25'), (26').

Of course, we cannot conclude from this that the acceleration of the gas affects only the arc column, since, as pointed out above, the exponent of the given criterion depends strongly on the heater operating regime.

It should be noted that the complex G/d_c does not enter into expression (27'). This is as it should be, since in segmented electrodes the entire channel is occupied by the arc, through which the gas is blown. Therefore, the Peclet number, reflecting the heating of the gas by the heat flux, has no significance. In expressions (25) and (26), on the other hand, since they cover experiments with a weak flow over the arc, it is desirable to introduce the Pe number. In this case, the accuracy of the formulas should increase.

To sum up, we may conclude that the process of generalization of the current-voltage characteristics of electric arcs in gas flows is still far from complete. We still lack generally recognized reliable formulas. The use of the methods of approximate similarity requires great skill in selecting the principal dimensionless arguments. As experimental data have accumulated, investigators have tried out various criteria corresponding to their particular conception of the phenomenon. Some of these attempts must be acknowledged to have been unsuccessful; others, on the other hand, are quite promising. It is reassuring that it has already proved possible to establish a number of important criteria and to some extent circumvent the difficulty of detemining the scales of reference of the physical properties. On this basis, fairly simple generalized current-voltage characteristics have been obtained for arcs in a variety of gas flows. The accuracy of these formulas still leaves something to be desired, but could be raised by improving the methods of approximation and introducing certain additional criteria.

NOTATION

I—current
U—voltage
E—electric field strength
j—current density;
w—velocity,
G—gas flow rate
P—pressure
B—magnetic induction
T—temperature
ρ—density
σ—electrical conductivity
h—enthalpy
c_p—specific heat at constant pressure
μ_0—magnetic permeability
k—Boltzmann's constant

R—gas constant;
L—characteristic dimension;
Π_1–Π_{12}, Re, Pr, Pe, Kn, Eu—criteria;
D and d—diameters of the electrodes (large and small) or of the arc;
l—length (of electrode, arc, interelectrode insert, etc.);
M—molecular weight.
Subscripts: 0—scale of reference; c—cathode; a—arc; in—heater inlet; $i = 1,2,3,\ldots$

REFERENCES

1. S. S. Kutateladze and O. I. Yas'ko, IFZh, **7**, no. 12, 1964.
2. G. Yu. Dautov, M. F. Zhukov, and V. Ya. Smolyakov, PMTF, no. 6, 1961.
3. O. I. Yas'ko, IFZh, 7, no. 12, 1964.
4. F. Burhorn, H. Maecker, and T. Peters, Z. Phys., **131**, 28, 1951.
5. A. M. Zalesskii and G. L. Kikekov, Trudy LPI, no. 1, 1954.
6. G. Yu. Dautov and M. F. Zhukov, PMTF [Journal of Applied Mechanics and Technical Physics], no. 2, 1965.
7. A. E. Sheindlin, E. I. Asinovskii, V. A. Baturin, and V. M. Batenin, ZhTF, **33**, no. 10, 1963.
8. G. Yu. Dautov and M. F. Zhukov, PMTF [Journal of Applied Mechanics and Technical Physics], no. 2, 1965.
9. B. D. Voronin, A. M. Tsirlin, and M. Ya. Smelyanskii, Izv. SO AN SSSR, ser. tekhn. nauk, no. 10, 3, 1966.
10. B. D. Voronin, A. M. Tsirlin, and M. Ya. Smelyanskii, IFZh [Journal of Engineering Physics], **10**, no. 3, 1966.
11. B. D. Voronin, A. M. Tsirlin, and M. Ya. Smelyanskii, Khim. prom., no. 7, 1967.
12. B. D. Voronin, B. V. Zolotov, M. Ya. Smelyanskii, A. M. Tsirlin, and V. P. Tsishevskii, Elektrotermiya, no. 5, 1963.
13. G. Yu. Dautov, Yu. S. Dudnikov, and M. I. Sazonov, Izv. SO AN SSSR, ser. tekhn. nauk, no. 10, 3, 1965.
14. N. M. Belyanin, Izv. SO AN SSSR, ser. tekhn. nauk, no. 10, 3, 1966.
15. O. I. Mironov, Izv. SO AN SSSR, ser. tekhn. nauk, no. 3, 1, 1967.
16. G. P. Stelmakh, N. A. Chesnokov, and V. A. Sologub, Izv. SO AN SSSR, ser. tekhn. nauk, no. 3, 1, 1967.
17. S. S. Kutateladze, A. I. Rebrov, V. N. Yargin, PMTF, no. 1, 1967.
18. V. S. Kisel, B. A. Uryukov, and V. I. Yadrov, Izv. SO AN SSSR, ser. tekhn. nauk, no. 3, 1, 1967.
19. V. Ya. Smolyakov, PMTF [Journal of Applied Mechanics and Technical Physics], no. 1, 1967.
20. A. I. Zhidovich, O. A. Savel'ev, and O. I. Yas'ko, IFZh [Journal of Engineering Physics], **11**, no. 5, 1966.
21. A. B. Bublievskii, A. I. Zhidovich, S. K. Kravchenko, L. A. Shimanskii, and O. I. Yas'ko, Proc. Inter-Inst. Conf. on Chem. and Phys. of Low-Temp. Plasma [in Russian], Moscow State University, May 1967.
22. A. F. Bublievskii, S. K. Kravchenko, L. A. Shimanskii, and O. I. Yas'ko, Proc. Third All-Union Conf. on Low-Temp. Plasma Generators [in Russian], Minsk, Institute of Heat and Mass Transfer AS BSSR, June 1967.
23. E. A. Borovchenko, V. I. Krylovich, and O. Y. Yasko, "Some Problems on Heat Transfer in Plasma Jet Generators," Third International Heat Transfer Conference, Chicago, August, 1966.
24. F. B. Yurevich, M. V. Volk-Levanovich, and A. G. Shashkov, IFZh [Journal of Engineering Physics], **12**, no. 6, 1967.
25. V. L. Sergeev, IFZh [Journal of Engineering Physics], **9**, no. 5, 1965.
26. G. Yu. Dautov, M. F. Zhukov, A. S. Koroteev, V. Ya. Smolyakov, Yu. I. Sukhinin, and O. I. Yas'ko, in: Low-Temperature Plasma [in Russian], Mir, Moscow, 1967.
27. A. I. Zhidovich, S. K. Kravchenko, and O. I. Yas'ko, IFZh [Journal of Engineering Physics], **13**, no. 3, 1967.
28. J. K. Harvey, P. G. Simpkins, and B. D. Adcock, AIAA, **1**, no. 3, 1963.
29. H. Tateno and K. Saito, Japan J. Appl. Phys., 2, no. 3, 1963.
30. V. Ya. Smolyakov, PMTF, no. 6, 1963.
31. A. S. Koroteev, and O. I. Yas'ko, IFZh [Journal of Engineering Physics], **10**, no. 1, 1966.
32. A. I. Zhidovich, S. K. Kravchenko, and O. I. Yas'ko, Proc. Third All-Union Conf. on Low-Temperature Plasma Generators [in Russian], Minsk, Institute of Heat and Mass Transfer AS BSSR, June 1967.
33. A. I. Zhidovich, IFZh [Journal of Engineering Phsyics], **15**, no. 1, 1968.
34. L. I. Kolonina, and V. Ya. Smolyakov, Proc. Third All-Union Conf. on Low-Temp. Plasma Generators [in Russian], Minsk, Institute of Heat and Mass Transfer AS BSSR, June 1967.

35. G. Yu. Dautov and M. I. Sazonov, Proc. Third All-Union Conf. on Low-Temp. Plasma Generators [in Russian], Minsk, Institute of Heat and Mass Transfer AS BSSR, June 1967.

36. I. G. Panevin, P. P. Kulik, I. P. Nazarenko, B. P. Rychkov, and V. K. Tyulin, ibidem.

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